Name: ______________________________

Instructor: __________________________

Math 10120, Exam II
October 11, 2016

- The Honor Code is in effect for this examination. All work is to be your own.
- You may use a calculator .
- The exam lasts for 1 hour 15 minutes .
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 12 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

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Multiple Choice	______________
11.	______________
12.	______________
13.	______________
Total	______________

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Multiple Choice

1.(6 pts) A sample space consists of 7 simple outcomes $\{a, b, c, d, e, f, g\}$. The probabilities are

P(a)	P(b)	P(c)	P(d)	P(e)	P(f)	P(g)
0.11	0.20	0.15	0.30	0.09	0.05	0.1

What is $\mathrm{P}\big(\{c, g, a\}\big)$?

Solution: $\mathrm{P}\big(\{c, g, a\}\big) = P(c) + P(g) + P(a) = 0.15 + 0.11 + 0.1 = 0.36$

(a) 0.36 (b) 0.71 (c) 0.437 (d) 0.25 (e) $\frac{3}{7}$

2.(6 pts) Let E and F be events where $\Pr(E') = \frac{3}{5}$, Pr(F) $= \frac{4}{5}$, and $\Pr(E \cap F) = \frac{3}{10}$. Find $\Pr(E \cup F)$.

Solution: We have $P(E \cup F) = P(E) + P(F) - P(E \cap F)$.

$$P(E) = 1 - P(E') = 1 - \frac{3}{5} = \frac{2}{5}.$$

Therefore $P(E \cup F) = \frac{2}{5} + \frac{4}{5} - \frac{3}{10} = \frac{6}{5} - \frac{3}{10} = \frac{9}{10}$.

(a) $\frac{9}{10}$ (b) $\frac{2}{5}$ (c) 1 (d) $\frac{1}{2}$ (e) $\frac{3}{10}$

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3.(6 pts) Let E and F be events in a sample space with $Pr(E) = 0.5, \quad Pr(F) = 0.3$ and $Pr(E \cup F) = 0.7$. What is $Pr(E|F)$?

Solution: $Pr(E|F) = \dfrac{P(E \cap F)}{P(F)}$. To find $P(E \cap F)$ we use the inclusion exclusion principle: $P(E \cup F) = P(E) + P(F) - P(E \cap F)$. Plugging in the given probabilities, we get $0.7 = 0.5 + 0.3 - P(E \cap F)$. Therefore $P(E \cap F) = 0.1$. Now we have

$$Pr(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{0.1}{0.3} = \frac{1}{3}.$$

(a) $\frac{1}{3}$ (b) $\frac{1}{5}$ (c) $\frac{2}{3}$ (d) $\frac{3}{5}$ (e) $\frac{1}{2}$

4.(6 pts) A factory produces fuses, which are packaged in boxes of 20. A sample of five fuses is selected at random from each box for inspection. The box is rejected if at least one of these five fuses is defective. What is the probability that a box containing seven defective fuses will be rejected?

Solution: $P(\text{box is rejected}) = P(\text{at least one bad fuse is sample}) = 1 - P(\text{all good})$

$$= 1 - \frac{C(13,5)}{C(20,5)}.$$

(a) $1 - \dfrac{C(13,5)}{C(20,5)}$ (b) $1 - \dfrac{C(7,5)}{C(20,5)}$ (c) $\dfrac{C(7,1)C(13,4)}{C(20,5)}$

(d) $1 - \dfrac{7}{C(20,5)}$ (e) $\dfrac{C(7,5)}{C(20,5)}$

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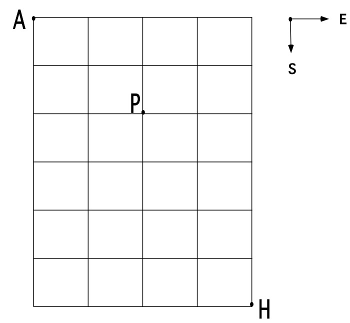
5.(6 pts) The map below shows the roads in a country town. Erin travels from A to H. She chooses a route at random from all routes that go South or East on every block. What is the probability that Erin passes through the intersection at **P**? (Probabilities are rounded to three decimal places.)



Solution: $\text{P(Erin passes thru P)} = \dfrac{\text{Number of routes Through P}}{\text{Total Number of routes from A to H}}$

$$= \frac{\text{(Number of routes From A to P)} \cdot \text{(Number of routes From P to H)}}{\text{Total Number of routes from A to H}}$$

$$\frac{C(4,2)C(6,2)}{C(10,4)} = \frac{6 \cdot 15}{210} = 0.4286.$$

(a) 0.429 (b) 0.1 (c) 1 (d) 0.9 (e) 0.571

6.(6 pts) Suppose an urn has 20 marbles in it, of which 10 are red, 6 are blue, and 4 are green. Suppose my experiment is to draw three marbles at random from the urn without replacement, and record the observed colors. Given that the 1st two marbles are red, what is the probability that the last marble is green?

Solution: If two red marbles have been drawn from the urn there are 8 red, 6 blue and 4 green marbles left in the urn. Therefore the probability that a green will be drawn given that the first two were red is $\frac{4}{18}$.

(a) $\frac{4}{18}$ (b) $\frac{4}{20}$ (c) $\frac{8}{20}$ (d) $\frac{6}{18}$ (e) $\frac{10}{18}$

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7.(6 pts) An electronic device contains 6 transistors operating independently of each other. The probability that a transistor will fail within 5 years is 0.02. What is the probability that at least one of the transistors will fail within 5 years?

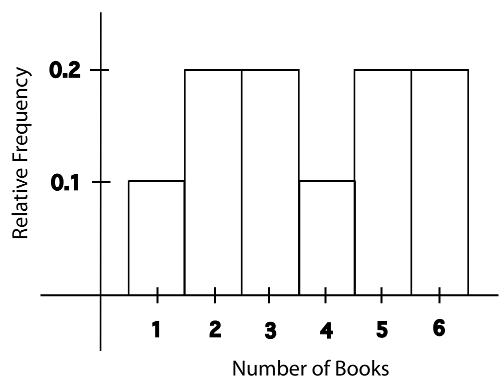
Solution: $P(\text{at least one will fail}) = 1 - P(\text{none will fail})$. By independence, $P(\text{none will fail}) = (0.98)^6$ and thus

$$P(\text{at least one will fail}) = 1 - (0.98)^6.$$

(a) $1 - (0.98)^6$ (b) $(0.98)^6$ (c) 1

(d) $1 - (0.02)^6$ (e) $(0.02)^6$

8.(6 pts) In a survey on the campus of the University of Mathland 300 students were asked to count the number of books in their backpack. The histogram below shows the results of the survey, giving the numbers recorded and their relative frequencies.



How many students in the sample had at least three books in their backpack at the time of the survey?

Solution: The proportion of students who have at least three book is the sum of the proportions of students with 3, 4, 5 and 6 books respectively; that is $0.2 + 0.1 + 0.2 + 0.2 = 0.7$. Since there were 300 students surveyed, the number with at least 3 books is

$$0.7(300) = 210.$$

(a) 210 (b) 7 (c) 200 (d) 150 (e) 3

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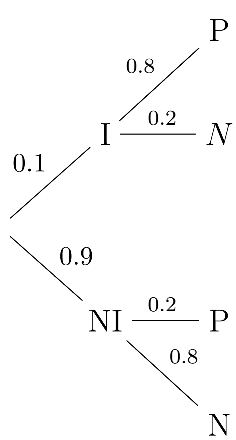
9.(6 pts) A test for mad squirrel disease (CJD) is 80% accurate, that is:

- when the test is applied, the results are either positive (has disease) or negative (does not have the disease).
- 80% of squirrels who have the disease test positive and
- 80% of squirrels who do not have the have the disease test negative.

The statistics show that 10% of the squirrel population in St. Liam County have CJD. A squirrel chosen at random tested positive for the disease. what is the probability that the chosen squirrel has CJD given that he tested positive?
(A tree diagram will almost certainly help.)

Solution: In the following diagram; I denotes infected, NI denotes not infected, P denotes that the squirrel tests positive (test indicates that the squirrel has the disease), N denotes that the squirrel tests negative.



We want to find $P(I|P) = \dfrac{P(I \cap P)}{P(P)}$.
From the diagram, we get

$$P(I|P) = \frac{(0.1)(0.8)}{0.1)(0.8) + (0.9)(0.2)} = 0.307.$$

(a) $\dfrac{8}{26}$ (b) 1 (c) 0.2 (d) $\dfrac{18}{26}$ (e) 0.8

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10.(6 pts) An experiment consists of drawing **a sample of 5 cards** from a standard deck of 52 cards. The sample space for this experiment is the set of all poker hands. (Recall that a standard deck of 52 cards has 4 cards from each denomination, Aces, Kings, Queens, Jacks, Tens, Nines, $\ldots$)

Let E be the event that the sample drawn has exactly 2 kings and let F be the event that the sample drawn has exactly 1 queen. How many outcomes are in the event $E \cap F$?

Solution: The event $E \cap F$ is the event that the sample contains exactly 2 kings and exactly one queen. The number of samples of this type is

$$C(4,2) \cdot C(4,1) \cdot C(44,2) = 22,704.$$

(a) 22,704 (b) 48 (c) 24 (d) 1584 (e) 1

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Partial Credit

You must show your work on the partial credit problems to receive credit!

11. (12 pts.) An experiment consists of flipping a coin 4 times and counting the number of heads.

(a) Complete the probability distribution table for this experiment given below.

Outcome	Probability
0	
1	
2	6/16
3	
4	

Solution:

Outcome	Probability
0	$\frac{C(4,0)}{2^4}$
1	$\frac{C(4,1)}{2^4}$
2	6/16
3	$\frac{C(4,3)}{2^4}$
4	$\frac{C(4,4)}{2^4}$

Outcome	Probability
0	$\frac{1}{16}$
1	$\frac{4}{16}$
2	6/16
3	$\frac{4}{16}$
4	$\frac{1}{16}$

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(b) What is the probability of getting at least two heads in this experiment?

$$P(\text{at least two heads}) = P(\text{exactly } 2)+P(\text{exactly } 3)+P(\text{exactly } 4) = \frac{6}{16}+\frac{4}{16}+\frac{1}{16} = \frac{11}{16}.$$

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12. (14 pts.) A total of 100 students and faculty at the University of Mathland were asked if they thought that a parking garage should be built on their campus. The results of the survey are shown below.

	Yes	No
Student	15	35
Faculty	40	10

An experiment consists of randomly selecting the records of the survey for one of the individuals surveyed.
Let S denote the event that the individual selected is a student,
Let F denote the event that the individual selected is a faculty member,
let Y denote the event that the answer of the individual selected was "Yes"
and let N denote the event that the answer of the individual selected was "No".

Solution: Taking totals, we get

	Yes	No	Total
Student	15	35	50
Faculty	40	10	50
Total	55	45	100

(a) What is the probability that the individual selected answered "Yes"?
(that is, what is Pr(Y) ?)

Solution: $P(Y) = \dfrac{55}{100}$.

(b) Given that the person selected is a student, what is the probability that they answered "Yes"? (that is, what is $P(Y|S)$?)

Solution: $P(Y|S) = \dfrac{\#(Y \cap S)}{\#(S)} = \dfrac{15}{50}$.

(c) Are the events Y and S independent?
Give a reason for your answer.
Solution: No because $P(Y) \neq P(Y|S)$.
(d) Are the events Y and S mutually exclusive?
Give a reason for your answer.
Solution: No because $P(Y \cap S) = \dfrac{15}{100} \neq 0$.

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13. (14 pts.) You have 3 left mittens and 2 right mittens in a box. In an experiment you draw mittens one at a time at random from the box without replacement.

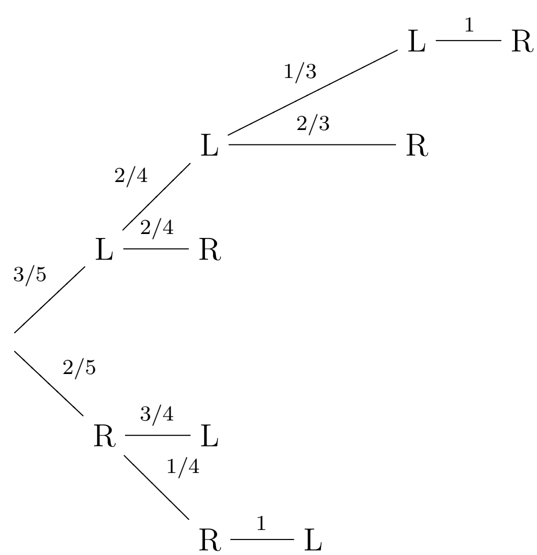
- You continue to select mittens until you have taken at least one right and one left mitten, then you stop and record the total number of mittens you have drawn from the box.

(a) What is the Sample Space for this experiment?
(Hint: Check the underlined sentence above.)

Solution You must take out at least 2 mittens to get a left and right mitten. After you have taken 4 mittens from the box, you must have one left and one right. So the sample space is

$$\{2, 3, 4\}.$$

(b) Use a tree diagram to determine the probabilities in the probability distribution for this experiment below.



(c) Complete the probability distribution for this experiment

Number of Mittens Drawn	Probability

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Solution:

Number of Mittens Drawn	Probability
2	$\frac{2}{5} \cdot \frac{3}{4} + \frac{3}{5} \cdot \frac{2}{4} = \frac{12}{20}$
3	$\frac{3}{5} \cdot \frac{2}{4} \cdot \frac{2}{3} + \frac{2}{5} \cdot \frac{1}{4} = \frac{18}{60}$
4	$\frac{3}{5} \cdot \frac{2}{4} \cdot \frac{1}{3} = \frac{6}{60}$

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Rough Work

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Instructor: ANSWERS

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3.	(•)	(b)	(c)	(d)	(e)
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5.	(•)	(b)	(c)	(d)	(e)
6.	(•)	(b)	(c)	(d)	(e)
7.	(•)	(b)	(c)	(d)	(e)
8.	(•)	(b)	(c)	(d)	(e)
9.	(•)	(b)	(c)	(d)	(e)
10.	(•)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice ______________

11. ______________

12. ______________

13. ______________

Total ______________